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Author(s): Miguel A. Kiguel and Andres Dauhajre
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# A DYNAMIC MODEL OF THE OPEN ECONOMY WITH SLUGGISH OUTPUT*

By Miguel A. Kiguel and Andres Dauhajre[1]

## 1. INTRODUCTION

Under a flexible exchange rate system, large fluctuations in the nominal and the real exchange rates seem to be the rule rather than the exception. The significant appreciation of the dollar during the early 1980's followed by its recent steep depreciation provides new evidence for this fact. The seminal work of Dornbusch (1976) introduces the notion that exchange rates are likely to be very volatile in the presence of sticky prices, a fixed level of output, and continuous adjustment in the asset markets. As a matter of fact, the assumption of instantaneous adjustment in the asset markets and the existence of sticky prices were central to overshooting the exchange rate in Dornbusch's framework. In that model, however, overshooting might not occur when output is allowed to adjust instantaneously in response to changes in aggregate demand.

The characterization of the adjustment mechanism in the goods market is essential to a thorough understanding of the dynamics of the exchange, prices, and output.[2] This is certainly the case in Dornbusch (1976), but it is also true in other models such as Obstfeld (1982), where overshooting will not occur in the presence of sticky wages when the goods market continuously clears. One of the reasons overshooting does not occur in Dornbusch and Obstfeld is the instantaneous adjustment of the level of output to the desired level (output is demand determined in Dornbusch (1976), while it depends on the interaction of demand and supply in Obstfeld (1982)). Nevertheless, this result is not exclusive to models in which the goods market continuously clears. For example, Levin (1985) showed that overshooting need not occur in Dornbusch's model when output responds sluggishly to excess demand pressures.

The purpose of this paper is to investigate the behavior of the exchange rate and the adjustment of the economy in response to monetary disturbances under alternative assumptions regarding the adjustment of the goods market. Through-

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[2] The structure of the goods market is extremely simplified in most models of exchange rate dynamics, by having output either exogenous and fixed, or alternatively demand determined. This is the case, for example, in Kouri (1976) Calvo and Rodriguez (1977), Frenkel (1978), Dornbusch and Fischer (1980), Rodriguez (1980), Obstfeld (1981), Mussa (1982), Driskill and McCafferty (1985), and Engel and Flood (1985) among others.

out the paper, we assume that prices are sticky and that there is a lag in the adjustment of the level of output.[3] In addition, we explicitly model the supply side of the economy and assume that the equilibrium level of output is determined through the interaction of aggregate supply and aggregate demand. The specification of the aggregate supply function and of the output adjustment mechanism is sufficiently broad to allow us to discuss cases in which a depreciation of the real exchange rate can have expansionary or contractionary effects on domestic output.

Though most works in open macroeconomics assume that a depreciation of the real exchange rate will have an expansionary effect on domestic output, Diaz Alejandro (1963), Cooper (1971), Taylor (1979), Krugman and Taylor (1983), and Calvo (1983), among others, show that a depreciation of the real exchange rate can be contractionary.[4] The dynamic properties of contractionary devaluation models are still the subject of wide research. Under flexible exchange rates, as discussed by Calvo (1983), there will be multiple equilibria when the system is stable. On the other hand, Calvo (1987) shows that when the monetary authorities follow a nominal exchange rate rule for reasonable values of the parameters, the system will have a unique stable solution. Our model establishes the conditions for flexible exchange rates under which the system will have a unique (saddle path) stable equilibrium. We also analyze whether the exchange rate overshoots its long-run equilibrium level.

The paper is organized as follows. In Section 2, we present the basic structure of the model. In many respects the basic analytical framework closely resembles Dornbusch (1976). We assume a small open economy under perfect capital mobility in which the interest rate parity condition holds. Assets markets continuously clear and agents have rational expectations regarding the paths of nominal and real variables. Aggregate demand for domestic output depends on real money balances and on the real exchange rate, and prices are sticky. One distinctive feature of this model lies on the characteristics of the supply function. In the long run, the desired supply of output is assumed to depend on relative prices. An increase (a reduction) in the relative price of the domestic good will lead to a higher (lower) level of output. In the short run, however, the desired output supply is responsive to changes in relative prices and in excess demand pressures.

[3] This assumption is consistent with recent work by Buiter and Miller (1983) and Levin (1985) and (1986) who postulate a similar adjustment mechanism in the goods market. In addition, Bhandari (1983) also introduces a slow adjustment in the goods market. However, in that model output remains fixed and aggregate demand adjusts sluggishly.

[4] Many of these papers were written for a fixed exchange rate system. As a result, this negative link between output and the real exchange rate is sometimes referred to as the case of contractionary devaluation. Though this terminology is not totally appropriate in our model, given that the exchange rate is flexible, we will nevertheless use it in later sections as a substitute for the case of contractionary real depreciation, since we believe that the term "contractionary devaluation" is the most representative of the literature on this subject.

Finally, we add a quote of realism by assuming that there is lagged output adjustment.[5]

Section 3 presents a discussion of the dynamic adjustment of the economy under alternative assumptions regarding the characteristics of the supply function. Our objective is to describe the adjustment of output, the exchange rate, and prices for the cases in which a depreciation of the real exchange rate has expansionary and contractionary effects. As a result, the present paper can also be viewed as an effort to develop a generalized framework to study the adjustment of the open economy, including the cases of expansionary and contractionary depreciation of the real exchange rate as special ones. We find that in the expansionary depreciation case overshooting of the exchange rate is likely to occur for reasonable values of the parameters in the presence of sluggish output adjustment. In addition, overshooting is even more likely to occur in the "contractionary devaluation" case. Finally, we show that the explicit introduction of the contractionary depreciation case is particularly relevant to understand fluctuations in output and in the real exchange rate of the type recently described by Edwards (1986). We state some final remarks in Section 4.

## 2. THE MODEL

*The Basic Framework.* In this section, we present the basic model and describe the equilibrium conditions for the goods and asset markets. We also derive the dynamic behavior of domestic prices, domestic output, and the exchange rate, and discuss the stability properties of the model.

We assume a standard demand for money function that depends positively on the level of domestic output ($y$) and negatively on the nominal interest rate ($i$). The supply of money ($m$) is fixed by the monetary authority and is exogenous to the model. The money market equilibrium condition can be written in log-linear form as

$$m - p = -\lambda i + \varphi y \tag{1}$$

where ($p$) is the price of the domestic good and where $\lambda$ and $\varphi$ are positive constants. Domestic and foreign assets are perfect substitutes for each other and, hence, the expected rate of return on these assets must be equalized (i.e. the interest rate parity condition continuously holds);

$$i = i^* + \dot{e}^e \tag{2}$$

where $i^*$ is the nominal rate of return on foreign bonds and $\dot{e}^e$ is the expected rate of depreciation of the exchange rate. In accordance with most of the recent literature, we will assume rational expectations (which in the present framework

[5] Blinder (1982) shows that firms might only partially adjust prices and output in response to demand shocks. In particular, when output is storable, following a temporary demand shock the market is likely to equilibrate mainly through changes in inventories, rather than in prices and the level of output.

is equivalent to perfect foresight). Hence the expected rate of depreciation of the exchange rate will always equal the actual one (i.e. $\dot{e} = \dot{e}^e$).

It is further assumed that the money market continuously clears. Substituting equation (2) into (1), using the rational expectations assumption and rearranging terms, we obtain the following equilibrium condition for the rate of depreciation of the exchange rate:

$$\dot{e} = (p - m - \lambda i^* + \varphi y)/\lambda. \tag{3}$$

The economy produces only one good which can either be consumed domestically or traded in the world markets. It also imports a foreign good, whose international price is given, and is an imperfect substitute for the domestic good. The economy is "large" in the market for the domestic good in the sense that its equilibrium price will depend on the prevailing supply and demand conditions. Since domestic prices are sticky, a depreciation of the exchange rate will affect relative prices.[6] The demand for the domestic good in log-linear form can be written as

$$y^d = \delta(e - p) + \sigma(m - p) \tag{4}$$

where $\delta$ and $\sigma$ are positive constants and where $(e)$ is the log of the nominal exchange rate. As usual, a depreciation of the real exchange rate $(e - p)$ increases the demand for domestic output as does an increase in real money balances.[7]

Following Dornbusch (1976) domestic prices are assumed to be sticky in the short run while they adjust over time in response to the excess demand pressures in the market for the domestic good. In particular, it will adjust according to the following rule[8]:

$$\dot{p} = \pi[\delta(e - p) + \sigma(m - p) - y] \tag{5}$$

where $\pi$ is a positive constant that denotes the speed of adjustment of domestic prices.[9]

We will now introduce the supply side of the economy. We assume that pro-

[6] The results of the present model could easily be extended for a country that produces tradeable and nontradeable goods. If that were the case then total output would be a weighted average of the production of tradeables and nontradeables. However, our main conclusions will not be affected as long as the non-traded sector is relatively large, or alternatively, as long as the price elasticity of the supply of output is greater in the nontradeable than in the tradeable sector. The latter condition would certainly be satisfied for many semindustrialized countries whose exports of primary products are, at least in the short run, price inelastic.

[7] The aggregate demand function in general also depends on real income. As the reader could verify when the system is stable its introduction will not affect the qualitative results presented in this paper. We decided to leave this effect out of the demand function in order to enhance the analytical tractability the model.

[8] The empirical works of Gordon (1982), Rotemberg (1982) and Meese (1984) strongly support the sticky price assumption.

[9] By assuming this price equation we are subject to some of the critcisms raised by Wilson (1979), Mussa (1982) and which have been recently discussed by Obstfeld and Rogoff (1984) regarding anticipated policy changes. However, since in this paper we only focus on unanticipated policy changes these criticisms do not represent an important limitation in our analysis.

ducers have a notional supply function that depends on the real exchange rate and on the excess demand pressures,[10]

(6) $$y^s = (1 - \alpha)\theta(p - e) + \alpha\omega[\delta(e - p) + \sigma(m - p) - y],$$

where $\theta > 0$, $1 \geq \alpha \geq 0$, $0 \geq \omega \geq 1$, and where $y^s$ is the amount of output that firms would like to produce in the absence of adjustment costs. Equation (6) departs from the standard assumptions in modeling open economies and, thus, requires further elaboration. In the long run, the goods market clears and, hence, aggregate supply is entirely determined by relative prices. In the short run, however, desired output also depends on excess demand pressures. The first term in equation (6) ($(1 - \alpha)\theta$) measures the direct change in desired output in response to a change in relative prices, sometimes called the Krugman-Taylor effect.[11] The second term in (6) introduces the excess demand effect emphasized in the literature on exchange rate dynamics. In the presence of an excess demand for the domestic good, firms will respond by increasing their production; the degree of this response is measured by the coefficient $\alpha\omega$. In our model, $(1 - \alpha)$ and $\alpha$ measure the weight that the firm attaches to the two arguments in the supply function.[12] When $\alpha$ is close to one, it can be argued that output is essentially demand determined. Alternatively, if $\alpha$ were close to zero, the desired amount of output ($y^s$) will essentially be determined by the Krugman-Taylor effect.

According to equation (6), when the economy is away from the stationary equilibrium, a change in relative prices has an ambiguous effect on the desired supply of output. While an improvement in the domestic relative price will directly induce higher desired output, it could also indirectly lead to its reduction through the effect on aggregate demand. The total effect of a change in relative prices on the desired supply of output will depend on the relative size of these two effects.

*The Dynamic Model.* We will now analyze the dynamic behavior of the economy. Equations (3) and (5) describe the motion of the exchange rate and domestic prices over time. In order to have a complete, dynamic specification of the model, we will introduce an equation that describes the evolution of output.

We assume that output does not adjust instantaneously to its desired level. The equation of motion of output can be written as[13]

(7) $$\dot{y} = \beta(y^s - y),$$

[10] For a complete discussion of the notional supply function see Barro and Grossman (1976).

[11] This type of supply behavior essentially corresponds to the one described by Krugman and Taylor (1978) in which the foreign good enters as an input in the production function of the domestic good. In that model, a devaluation increases the price of the imported input, and profit maximizing firms will reduce the level of supply.

[12] The use of linear weights to measure the significance of these two forces is going to be specially helpful for the interpretation of the results of this paper.

[13] This type of formulation is consistent with a large number of models that postulate that firms face costs in order to change the output levels, either due to costs in adjusting the stock of capital (as for example in Sargent (1981), chapter 14) and/or the labor services (see Phelps (1970)) used in production. In this type of framework, it is not in general optimal for them to move instantaneously to the level that would be optimal in the absence of such costs.

where $\beta$ is a positive parameter that denotes the speed of adjustment of output from its current to its desired level. Finally, substituting (6) into (7) yields:

$$\dot{y} = \beta(1 - \alpha)\theta(p - e) + \alpha\omega[\delta(e - p) + \sigma(m - p) + y] - y\}. \tag{8}$$

Equations (3), (5), and (8) determine a system of three differential equations in $e$, $p$, and $y$. We will now analyze the stability properties of the system and describe its motion using a version of the method used by Calvo (1985). The Jacobian of the dynamic system is given by

$$A \equiv \begin{pmatrix} 0 & 1/\lambda & \varphi/\lambda \\ \pi\delta & -\pi(\delta + \sigma) & -\pi \\ \beta[\alpha\omega\delta - (1 - \alpha)\theta] & \beta[(1 - \alpha)\theta - \alpha\omega(\delta + \sigma)] & -\beta(1 + \alpha\omega) \end{pmatrix} \tag{9}$$

The dynamic system can now be written as

$$\begin{pmatrix} \dot{\tilde{e}} \\ \tilde{p} \\ \tilde{y} \end{pmatrix} = A \begin{pmatrix} \tilde{e} \\ \tilde{p} \\ \tilde{y} \end{pmatrix} \tag{10}$$

where $\tilde{x} = x - \bar{x}$, $\bar{x}$ being the equilibrium value of the variable. In (10) there are two state variables, ($p$ and $y$), while ($e$) is free to jump. Consequently, the system will have a unique stable solution if and only if it has one positive and two negative characteristic roots. As can be noticed readily from (9), the trace of $A$ is negative while a sufficient condition for the determinant to be positive is

$$(1 - \sigma\varphi) \geq 0. \tag{11}$$

In Section 3, we assume that condition (11) is satisfied assuring stability and uniqueness.[14]

## 3. MACROECONOMIC ADJUSTMENT

In this section, we discuss the dynamic behavior of the economy. The analysis shows that the structure of the aggregate supply function and the presence of sluggish output adjustment are particularly relevant for a complete characterization of the motion of the economy. We analyze the cases of expansionary and contractionary "devaluation" and explore the circumstances under which overshooting of the exchange rate could arise in each of these cases. Finally, we study the conditions under which this type of model can generate fluctuations in output and the real exchange rate, and briefly discuss the plausibility of the dynamic pattern generated.

It is known, from Calvo (1987), that one can fully characterize the adjustment of the economy on the basis of the characteristic vectors of the system. This

[14] The values of the determinant and of the trace of $A$ are

$$|A| = (\pi\beta/\lambda)[\delta + (1 - \alpha)\theta(1 - \sigma\varphi)]$$

$$\text{tr (A)} = -\pi(\delta + \sigma) - \beta\omega(1 + \alpha) < 0.$$

method is particularly helpful in this model in which we have a system of three differential equations. It is convenient for the forthcoming discussion to concentrate our analysis on the phase diagram that shows the motion of the economy in the output-exchange rate space.[15] The corresponding characteristic vector is[16]

$$(12)\qquad x_3/x_1 = [\beta/(c + \beta(1 + \alpha\omega)w)] \times \{(\lambda c + \varphi c/\pi - 1)[(1 - \alpha)\theta - \alpha\omega\delta) + \alpha\omega\sigma\lambda c + (1 - \alpha)\sigma\varphi\theta\}.$$

Without any loss of generality, we assume that the denominator

$$(13)\qquad [c + \beta(1 + \alpha\omega)]w$$

is negative for the non-dominant characteristic root and positive for the dominant one (i.e., the one associated with the largest stable characteristic root);[17] $w$ is defined as

$$(14)\qquad w \equiv [1 - \varphi(\delta + \sigma) - (c\varphi/\pi)]$$

and is assumed to be positive for the case under study in which the characteristic roots ($c$) are negative.[18] On the other hand, the sign of the numerator is ambiguous and, as will be shown shortly, critically depends on the structure of the desired supply of output function.

In our discussion we take advantage of two properties of the trajectories of differential equations. First, it is known that the system will approach the steady state equilibrium along the dominant characteristic vector and, second, the trajectory cannot cross the non-dominant characteristic vector.[19] Following a large number of models in the rational expectations literature, we rule out speculative bubbles (i.e. unstable paths) and concentrate on the stable solutions of the system. In all the cases that we analyze, it is assumed that the economy is initially at its stationary equilibrium and that the monetary authority unexpectedly makes a once and for all increase in the money supply.

*Expansionary Depreciation.* Most works on open economy macroeconomics assume that a depreciation of the real exchange rate has an expansionary effect on domestic output. Those models incorporate the widely held view that a depreciation that successfully changes relative prices increases the demand for domestic

[15] The reader should be aware that the analysis could alternatively be carried out in the $p$-$y$ or $p$-$e$ planes. However, the $y$-$e$ plane appears to be the most convenient for our discussion.

[16] In the appendix we present the characteristic polynomial of the system and derive the three corresponding characteristic vectors.

[17] In appendix III (not included for publication) we prove that the dynamic behavior of the system is independent of the sign of equation (13).

[18] As the reader could verify, the assumptions regarding the signs of $w$ and the denominator do not constraint the results obtained in this paper.

[19] For a discussion of these results for the two by two system see Levin (1981). If the characteristic roots of the system are complex conjugates, then the system will portray an oscilatory movement towards the stationary equilibrium and hence the above result does not apply.

output through the substitution effect. As a result, in the neo-keynesian works that follow the Mundell-Fleming tradition, output is demand-determined and a depreciation of the exchange rate has an expansionary effect on output. In the present model, although a depreciation always increases the demand for domestic output, it has an ambiguous effect on the level actually produced. The effect of a devaluation will now be expansionary or contractionary depending on the values of the parameters of the aggregate supply and the aggregate demand functions and on the speeds of adjustment of prices and output.

A necessary and sufficient condition for a devaluation to be expansionary is that the numerator in equation (12) be positive. A sufficient condition for this result is

$$(1 - \alpha)\theta - \alpha\omega\delta < 0. \tag{15}$$

Condition (15) is closely related to the aggregate supply function introduced in equation (6). It will be negative when a depreciation in the *nominal* exchange rate increases the desired supply of output. In this case, output essentially behaves as if it were demand-determined; the "excess" demand effect ($\alpha\omega\delta$) dominates the cost effect ($(1 - \alpha)\theta$). It can also be readily noticed from (12) that when $\alpha = 1$ (i.e. the desired supply of output entirely depends on excess demand pressures) a depreciation will be expansionary even in the presence of sluggish output adjustment.[20]

One additional interesting aspect in this respect arises from the effect of domestic prices on desired supply. From the numerator in (12), we can observe that a depreciation of the *real* exchange rate can be expansionary even when condition (15) is not satisfied. A necessary (though not sufficient condition) for this to happen is

$$(1 - \alpha)\theta - \alpha\omega(\delta + \sigma) < 0. \tag{16}$$

Condition (16) not only incorporates the "direct" effect of a change in relative prices on desired supply (as presented in (15)), but also the "indirect" real balance effect that results from a change in prices. In other words, an increase (a fall) in the domestic price could reduce (increase) aggregate demand not only as a result of the lower (higher) relative price of the foreign good, but also due to the reduction (increase) in real money balances.

The motion of the system is portrayed in Figure 1. *DD* and *NN* are the dominant and non-dominant characteristic vectors that correspond to equation (12) when the numerator is positive. It can be noticed readily that the slope of the dominant characteristic vector is positive while the non-dominant is downward sloping. If we assume that the economy is initially at a point like *A*, the motion of the system towards the steady state will occur inside the first quadrant along the AFO path. Moreover, as the economy moves from *A* to *O*, output will remain above the full employment level; as a result, a depreciated real exchange rate has

[20] The works of Levin (1985) and Buiter and Miller (1983) only consider the expansionary devaluation case.

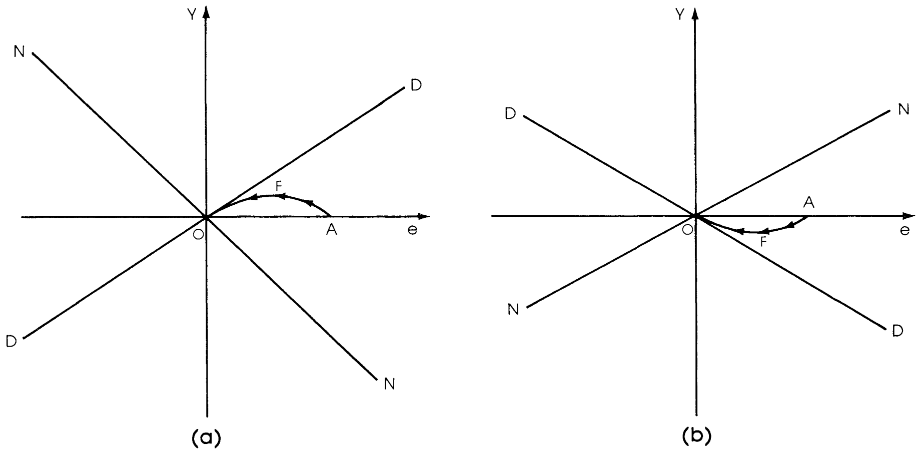


FIGURE 1

an expansionary effect on domestic output. The higher (lower) level of aggregate demand resulting from the depreciated (appreciated) real exchange rate increases (reduces) domestic output.

There is one additional and important aspect that emerges from this model. There is a widespread perception that time series data portrays a positive co-movement between output and prices. In Dornbusch (1976) and other similar models, in which prices are sticky and output demand determined, there is an initial increase in output following the devaluation of the exchange rate (this occurs in response to an increase in the money supply). Over time, though, as prices rise, the nominal (and real) exchange rate appreciates and output falls. As a result, the Dornbusch-like model implies that output and prices tend to move in opposite directions. In our model, we obtain a more plausible relationship between prices and output. Indeed, the introduction of sluggish output adjustment is crucial to generate this type of behavior.

Will we observe exchange rate overshooting in the presence of sluggish output adjustment? In Dornbusch (1976) there is overshooting when output is fixed, while overshooting might not occur when output is demand-determined and the income elasticity of the demand for money is "large." In the present model, there will be overshooting when the following condition is satisfied[21]:

$$\varphi(\delta + \sigma) < \delta/(\lambda\beta\omega\alpha(\delta + \sigma) + (1 + \alpha). \tag{17}$$

As in Dornbusch (1976), the smaller the income elasticity of money demand ($\varphi$)

[21] For a complete derivation of condition (17) the reader is refered to appendix IV (not intended for publication).

and the smaller the interest rate elasticity ($\lambda$) the more likely it is that overshooting will occur.[22] On the other hand, overshooting is less likely to occur the larger the speed of adjustment of output ($\beta$), the real balance elasticity of the demand for domestic output ($\sigma$), and the larger the "pure" elasticity of desired supply to excess demand pressures ($\omega$). The relative price elasticity of the demand for goods ($\sigma$) and the weight assigned to the excess demand pressures ($\alpha$) have an ambiguous effect on the initial response of the exchange rate. These results are essentially consistent with the ones found by Levin (1985), despite the use of a different adjustment mechanism for the goods market.

*Contractionary Depreciation.* The contractionary effects of a devaluation have been considered a distinctive possibility in a considerable number of theoretical and empirical works such as Calvo (1983), Cooper (1971), Diaz Alejandro (1967), Edwards (1986), Hanson (1979), Krugman and Taylor (1979), Levin (1981), and Taylor (1979). Further interest arose as a result of the stabilization programs in the late 1970's in the Southern Cone countries (Argentina, Chile, and Uruguay); the economies experienced a drastic overvaluation of the domestic currencies at the same time that there was an expansion in domestic output. Though most of the papers have studied this issue for economies under a fixed (or crawling peg) exchange rate system, many of the results of those models can be extended to a flexible exchange rate system.

We will now analyze the conditions under which a depreciation in the real exchange rate would have a contractionary effect on the level of domestic output in the present model, and discuss the resulting exchange rate and price dynamics. As was argued in the previous section, the characteristics of the adjustment of the economy, to a large extent, depend on the sign of the numerator of equation (12). We will now consider the case in which its value is negative. A sufficient condition for this is

$$(1 - \alpha)\theta - \alpha\omega(\delta + \sigma) > 0. \tag{18}$$

Condition (18) is also closely related to equation (6). It implies that, in response to an improvement (deterioration) in the relative price of the domestic good, firms will increase (reduce) their desired supply of output. This result will occur when the cost effect $(1 - \alpha)\theta$ in equation (6) (i.e. a Krugman-Taylor type effect) dominates the excess demand effect $\alpha\omega(\delta + \sigma)$ (i.e., the change in desired supply that occurs as a result of changes in aggregate demand). It should also be noticed that when aggregate demand plays no role in the determination of the desired supply of output (i.e., when $\alpha = 0$), the numerator in equation (12) will always be negative.

[22] In the present model we do not include disposable income as an argument in the aggregate demand function. If we had included it (as Dornbusch did) then condition (17) would become

$$\varphi(\delta + \sigma) < \delta/(\lambda\beta\omega\alpha(\delta + \sigma) + (1 + \alpha - \gamma)$$

where $\gamma$ is the marginal propensity to spend out of income. As in Dornbusch, the larger $\gamma$ the less likely overshooting would be.

When condition (18) is satisfied, the dominant characteristic vector (i.e., the one associated with the dominant characteristic root) will be downward sloping, while the non-dominant characteristic vector will have a positive slope. On the basis of these results, we can now analyze the dynamic behavior of the economy.

There are at least three important issues regarding the dynamic behavior of the economy in the present model. First, the system will always have a unique stable equilibrium path. In this respect, our results differ from Calvo (1983), where under flexible exchange rates and sticky prices (in his case, as a result of staggered contracts), when a devaluation is contractionary, there is either "a continuum of converging equilibria" or if not there is "none" (i.e. the system is unstable).[23] Second, as in Section 3, in response to an unanticipated increase in the money supply, the exchange rate is likely to overshoot its long-run equilibrium level. A sufficient condition for overshooting will now be[24]

$$(19) \qquad (1-\alpha)\theta - \alpha\omega(\delta+\sigma)(\lambda(C_1 C_2) - \varphi\beta((1-\alpha)\theta - \alpha\omega\delta)) + \alpha\omega\sigma\pi(\delta+\sigma) > 0.$$

Unfortunately, the interpretation of condition (19) is more cumbersome than that of (17) because the inequality depends on the values of the parameters of the model, and on the values of the characteristic roots ($C_1$ and $C_2$), which themselves are a function of the parameters of the model. According to condition (19), overshooting would become less likely the larger are $\beta$, $\varphi$, $\theta$, and $(1-\alpha)$. In addition, based on numerical experiments, we found that increases in $\theta$, $\varphi$ and/or $(1-\alpha)$ will actually reduce the size of the overshooting, while $\beta$ essentially does not have any effect on it. Moreover, the experiments that we performed also showed that when (18) is satisfied overshooting will occur for any plausible combination of values of the parameters. Actually, this result is entirely consistent with the structure of the present model when a devaluation is contractionary. In the previous section, when there is an unanticipated expansion in the money supply, overshooting might not occur if the demand for money increases as a result of a higher level of income. In the present case, however, in response to a monetary expansion, the level of output falls and, hence, at the old interest rate, there will be an excess supply of money. As a result, the interest rate falls and, given that the interest rate parity condition holds, the exchange rate must overshoot its long-run equilibrium level. Finally, the present case does generate an unambiguous result regarding the co-movements in output and prices. During the early stages of the adjustment, they tend to move in opposite directions, though this tendency is reversed as the economy moves closer to the stationary equilibrium.

A diagramatical representation of the dynamic system is shown in Figure 1.b where we portray the dominant and non-dominant characteristic vectors (*DD* and *NN* respectively) that correspond to the case in which the numerator in equation (12) is negative. Suppose now that the economy is initially at its stationary equilibrium (point *O*) and that there is an unanticipated increase in the

[23] Calvo (1985) establishes conditions for uniqueness for the the contractionary devaluation case under a fixed exchange rate system.

[24] This expression is also derived in appendix IV.

money supply. Given that both output and domestic prices are predetermined in the short run, the exchange rate is the only variable that can adjust (jump) in response to the new development. The exchange rate originally depreciates and, if (19) is satisfied, it will overshoot its long run equilibrium level, moving the economy to a point like $A$ in Figure 1.b. At point $A$, there will be an excess demand for the domestic good and, hence, domestic prices will start to rise. The real exchange rate will be depreciated and, with the cost effects dominating output determination, there will also be a disincentive for producers to increase the level of output (actually, in this case they would reduce production). Over time, as the domestic relative price improves, production increases, but output remains below its full employment level throughout the adjustment process. It is interesting to notice that in this case an expansionary monetary policy will lead to an initial reduction in the level of output. The unanticipated increase in the money supply raises aggregate demand and inflation, however, since the real exchange rate depreciates, the level of output falls. Consequently, during the early stages of the adjustment process, the economy experiences a stagflationary situation in which inflation is rising at the same time that output is falling.

Finally, it should be noticed that in the present model the contractionary effects of a devaluation are not derived from a change in income distribution, as in Diaz Alejandro (1963), nor from a reduction in real income, as in Taylor (1979). It neither depends on the sizes of the import and export elasticities of demand, as in Levin (1981), nor it is derived from a wealth effect, as in Calvo (1983). In our model, as in Krugman and Taylor (1983), it results from a deterioration in the relative price of the domestic good which, as one would expect, disincentives its production.

*Output and Real Exchange Rate Fluctuations.* Most countries that adopted a flexible exchange rate system experienced significant fluctuations in the real exchange rate. This behavior could have been the result of various independent disturbances that produce large (and opposite) movements in the real exchange rate (such as drastic changes in monetary policy) or, instead, the result of a dynamic adjustment that already embodies a cyclical behavior of the macroeconomic variables. While there is an important body of theoretical works that can be used to explain the former type of fluctuations, it would certainly be useful to analyze whether this type of behavior can arise in a perfect foresight model.[25]

The present model can generate a cyclical pattern in output and in the real exchange when the stable characteristic roots have an imaginary part. This is discussed in some detail in Appendix I where we show that cycles are more likely to arise the larger are $(1 - \alpha)$ and $\theta$, and the smaller are $\delta$ and $\sigma$. In other words, as the Krugman-Taylor effect becomes the dominant force in the aggregate supply function (i.e., it prevails over the excess demand effect) the possibility of economic fluctuations increases.

[25] Driskill and McCafferty (1985) are also able to generate fluctuations in economic variables by including wealth in the aggregate demand function. In their model, though, they only analyze the expansionary devaluation case.

There is a straightforward interpretation of this result. When the cost effect dominates the excess demand effect, prices and output tend to move in opposite directions. If, for example, the economy has a depreciated (an appreciated) real exchange rate, there will be an excess demand (supply) for the domestic good and, from equation (5), we can observe that domestic prices will be rising (falling). On the other hand, from equation (6), the depreciated (appreciated) exchange rate will cause a reduction (an increase) in output. This divergent motion of prices and output in response to excess demand pressures is the ultimate reason for the emergence of cycles in this type of scenario. A sufficient condition for the existence of cycles in this model is[26]

$$(\beta\varphi/\lambda)[(1-\alpha)\theta-\alpha\omega\delta] > (1+\alpha\omega)^2\beta^2+\pi^2(\delta+s)^2 + p\beta[(1+2\alpha)(\delta+\sigma)-(1-\alpha)\theta]+\delta\pi/\lambda. \quad (20)$$

According to this expression as $\alpha \to 0$, it is more likely that the economy will experience fluctuations in the real exchange rate. On the other hand, as $\alpha \to 1$, condition (20) is unlikely to be satisfied. In particular, when $\alpha = 1$, the right side of (20) will always be greater and there will be no cycles.

We can gain further insight by performing some numerical experiments with the model. We assigned values to the parameters based on the works of Taylor and Carlozzi (1985) and Buiter and Miller (1983). In particular, we assume the following (basic) values:

$$\alpha = .5 \quad \beta = 1 \quad \pi = 3 \quad \delta = .1$$
$$\lambda = 4 \quad \sigma = .25 \quad \varphi = .9 \quad \theta = .5 \quad \omega = 1.$$

These values appear to be consistent with most econometric estimations of similar models. Moreover, we have intentionally assumed $\pi > \beta$ as a way to incorporate the fact that price adjustments in general take place faster than output adjustments.

In Table 1, we present the basic values of the parameters and the ranges for which they will lead to fluctuations in the system. Table 1 does not include the values of $\varphi$ and $\lambda$ because they do not have any significant effect on the values of the characteristic roots, at least for the relevant range of values that we are considering.

Table 1 shows at least three important results. First, it is apparent that, for the basic values being considered, there will be economic fluctuations. Second, the results support the notion introduced in this section that cycles are more likely to arise as the cost effect becomes the dominant force in the aggregate supply function. Third, an increase in the adjustment speed of output reduces the likelihood of cycles even in the contractionary devaluation case. Finally, and quite surprisingly, changes in the speed of adjustment of prices have an ambiguous effect. Table 1 shows that, for the contractionary devaluation case, cycles will not occur for relatively high or relatively low speeds of price adjustment.

[26] For a complete derivation of condition (22) the reader is refered to appendix II.

TABLE 1

| Parameter | Basic Value | Range for which will Generate Cycles |
|---|---|---|
| $\alpha$ | 0.50 | $\alpha < 0.55$ |
| $\beta$ | 1.00 | $\beta < 1.70$ |
| $\theta$ | 0.50 | $\theta > 0.35$ |
| $\sigma$ | 0.25 | $\sigma > 0.55$ |
| $\pi$ | 3.00 | $10 > \pi > 2.0$ |
| $\delta$ | 0.10 | $\delta < 0.30$ |

In Figure 2 we show the results obtained from simulations following an unanticipated increase in the money supply. For the purpose of this simulation, we use the the basic values presented in Table 1. This exercise shows in this case that the exchange rate will overshoot its long-run equilibrium level, and that there will be fluctuations in prices, output, and the nominal and real exchange rate. A close look at the cycles shows that output and the real exchange rate tend to move in opposite directions; a devaluation has a contractionary effect on domestic output, while prices and output tend to move in the same direction. On the other hand, the simulations show that output could be above full employment at times when the exchange rate is also above its equilibrium level, indicating that a casual observation of output and the real exchange rate could be misleading in this case. Figure 2 also shows that the exchange rate fluctuations tend to be shorter and to portray a greater amplitude than the fluctuations in output.

The outcome of this simulation strikingly resembles the recent empirical work performed by Edwards (1986). Following a devaluation of the exchange rate, he found that there is, in general, an inital reduction in the level of output followed by an expansionary increase in later periods. Actually, as can be noticed from Figure 2, this is the type of output behavior predicted by our model following a depreciation in the real exchange rate.

## 4. FINAL REMARKS

This paper investigates the effects of introducing sluggish output adjustment on the dynamics of prices, output, and the exchange rate. Our purpose is to analyze whether this framework would generate co-movements in prices, output, and the exchange rate that are significantly different from the ones generated in models where output is either exogeneous and fixed or demand-determined. The analytical framework that we used is sufficiently general to allow us to discuss these issues for cases in which a depreciation of the exchange rate could have expansionary or contractionary effects on domestic output. We also investigate whether overshooting of the exchange rate is likely to occur in each of these cases.

The analysis indicates that, indeed, using our model, the co-movements in output, prices, and the exchang rate are different from the ones described for models in which output is demand-determined (e.g., appendix in Dornbusch (1976)). For example, we showed that when a devaluation is expansionary, during

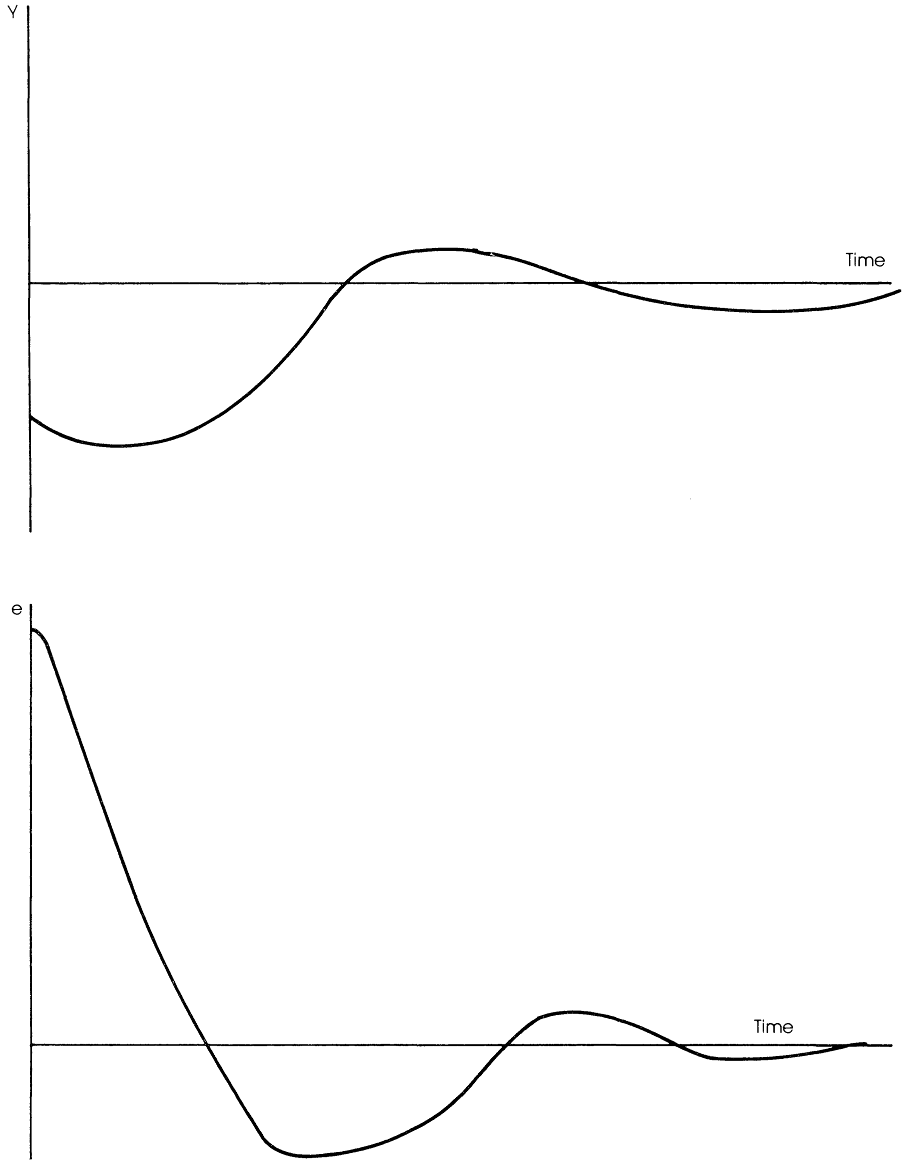


FIGURE 2

the early stages of the adjustment process, output and prices will rise while the exchange rate will appreciate. On the other hand, if output were entirely demand-determined, output and prices would move in opposite directions. When the devaluation has contractionary effects on domestic output, we were able to establish that there is a unique stable path describing the motion of macroeconomic variables. In addition, in this case, the system is also likely to generate overshooting of the exchange and fluctuations in output and in the real exchange rate.

The present paper is particularly useful to highlight the role of the supply side in models of exchange rate determination. Indeed, the results presented in this paper suggest, to a large extent, that they depend on the specification of the supply function. Whether a depreciation of the real exchange rate has expansionary or contractionary effects on domestic output, or whether output adjusts instantaneously or with a lag, is shown to be very important to characterize the macroeconomic adjustment of the economy. As a result, further efforts should be devoted to study problems of the open economies utilizing frameworks that explicitly incorporate the supply side, as well as the demand side, of the economy into the analysis.

*The World Bank, U.S.A.*
*Universidad Catolica, Dominican Republic*

## APPENDIX

In these appendices we discuss in more detail the properties of the dynamic system presented in equation (10). We present the characteristic polynomial of the system, derived the corresponding characteristic vectors and analyze the conditions under which the characteristic roots of the system are likely to have an imaginary part (i.e. the system will portray oscilations).

I. We can now calculate the characteristic roots of the system from the characteristic polynomial given by

$$(A.1)\qquad P(c) = -c^3 - [\pi(\delta + \sigma) + \beta(1 + \alpha\omega)]c^2 + \{(\beta\varphi/\lambda)[\alpha\omega\delta - (1 - \alpha)\theta] - \pi[\beta((1 - \alpha)\theta + \sigma + \delta) - \delta/\lambda]\}c + (\pi\beta/\lambda)[(1 - \alpha)\theta(1 - \sigma\varphi) + \delta]$$

Figure A.1 depicts the polynomial (PP) and its three characteristic roots (i.e., the values for which the polynomial is zero). It intersects the vertical axis on its positive side at the value of the determinant $|A|$. Moreover, since there are two negative and one positive characteristic roots, the basic configuration of the polynomial will unambiguously correspond to the one drawn in Figure A.1 (i.e., it is upward sloping at the point where it intersects the vertical axis, and it will be downward sloping when it crosses the positive side of the horizontal axis).

If the solution to (A.1) yields characteristic roots with an imaginary part, then the polynomial will correspond to the dotted line (P′P′) which does not intersect the horizontal axis on the negative quadrant.

The effects of the polynomial of changes in the values of various parameters of the model can be analyzed from the following equations

$$(A.2)\qquad \partial P(c)/\partial\alpha = \beta\{-\omega c^2 + [(\delta\omega + \theta)\varphi/\lambda + \pi\theta]c - (\theta\pi)/\lambda(1 - \sigma\varphi)\} < 0$$

$$(A.3)\qquad \partial P(c)/\partial\beta = -(1 + \omega\alpha)c^2 + \{(\varphi/\lambda)S - \pi[(1 - \alpha)\theta + \sigma + \delta]\}c + (\pi/\lambda)[\theta(1 - \alpha)(1 - \sigma\varphi) + \delta] \gtrless 0.$$

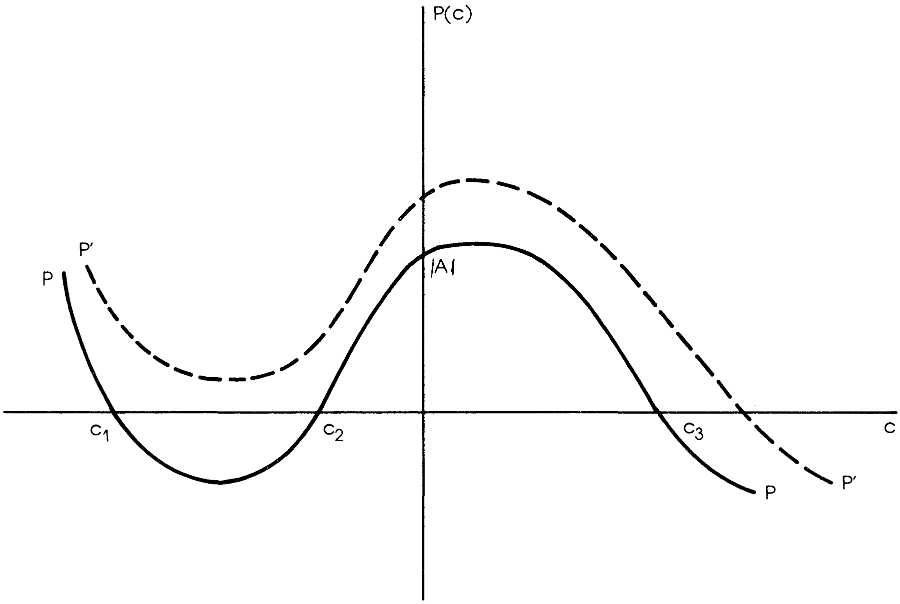


FIGURE A.1

(A.4) $$\partial P(c)/\partial \delta = -\pi(c + \beta)(c - 1/\lambda) + \beta\varphi\alpha\omega c\lambda \gtrless 0.$$

(A.5) $$\partial P(c)/\partial \sigma = -c(c + \beta) - [\beta\varphi(1 - \alpha)/\lambda) \gtrless 0$$

(A.6) $$\partial P(c)/\partial \theta = -[(\varphi/\lambda + \pi)\beta(1 - \alpha)]c + (\pi\beta/\lambda)(1 - \alpha)(1 - \sigma\varphi) > 0.$$

(A.7) $$\partial P(c)/\partial \pi = -(\delta + \sigma)c^2 + (1/\lambda) \times \{\beta(1 - \alpha)\theta[(1 - \sigma\varphi) - \lambda c] - [\beta\lambda\sigma + \delta(\lambda\beta - 1)]c + \beta\delta\} \gtrless 0.$$

where $S \equiv (1 - \alpha)\theta - \alpha\omega\delta$. It can be readily noticed from (A.2) and (A.6) that as the significance of the cost effect increases so does the the likelihood of having imaginary roots. Either a rise in the elasticity of the supply of output with respect to relative prices or a smaller weight for demand pressures on output determination would unambiguously shift the polynomial upwards and above the horizontal axis.

On the other hand, changes in the parameters of the demand function ($\delta$ and $\sigma$) or in the speed of adjustment of prices ($\pi$) or output ($\beta$) have an ambiguous effect on the value of the polynomial. However, the smallest characteristic root will satisfy

$$c + \beta < 0;$$

implying that the polynomial would move downward as either $\delta$ or $\sigma$ increase. It then becomes apparent that all roots will be real independently of whether the derivatives for the dominant characteristic root is positive or negative. On the other hand, one cannot derive any similar results for (A.2) or (A.6) which remain ambiguous.

The characteristic vectors associated with the stable characteristic roots can be calculated from matrix $A$ in equation (9) of the main body of the paper. We can write the characteristic equation as

$$Ax = c^*x \tag{A.8}$$

where $x$ is a $3 \times 1$ characteristic vector and $c^*$ is its corresponding characteristic root. As it is well known the system of equations (A.8) is linearly dependent and, hence, in order to solve we normalize it by making $x_3 = 1$. We then obtain the following relations

$$X_1/X_2 = w/[\lambda c^* - \delta\varphi] \tag{A.9}$$

$$X_3/X_1 = \beta\{\lambda[S - \alpha\omega\sigma]c^* + S\varphi c^*/\pi - S + h\}/(Dw) \tag{A.10}$$

$$X_3/X_2 = \beta\{\lambda[S - \alpha\omega\sigma]c^* + S\varphi c^*/\pi - S + h\}/D(\lambda c^* + \delta\varphi) \tag{A.11}$$

where

$$w \equiv (1 - \varphi(\delta + \sigma) - c^*\varphi/\pi);$$

$$h \equiv (1 - \alpha)\theta\sigma\varphi;$$

$$D \equiv c^* + \beta(1 + \alpha\omega);$$

and where $c^*$ is one of the stable characteristic roots. Notice that $X_3/X_1$ can be mapped in the $y$-$e$ plane, $X_3/X_2$ in the $y$-$p$ plane and $X_1/X_2$ in the $e$-$p$ plane.

II. We will now derive sufficient conditions for the system to have imaginary characteristic roots. The solution of the roots of a third degree polynomial of the form

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 0 \tag{A.12}$$

requires the solution of expressions which are quite cumbersome (for a complete presentation of the solution see for example the Encyclopedia of Mathematics). For our purposes, it is sufficient to note that one of the characteristic roots will be

$$x_3 = D - 1/3(a_2/a_3) = D - (1/3)t \tag{A.13}$$

where $t \equiv \pi(\sigma + \delta) + (1 + \alpha)\beta$, and where $D$ is a positive constant that depends on the parameters of the model. The solution of the other two roots can be find by solving

$$x_1, x_2 = -(D + (2/3)t) \pm \sqrt{(4/3)t^2 - 3d_2^2 + 4a_1} \tag{A.14}$$

PROPOSITION I. *When $X_3$ is positive it is possible to derive sufficient conditions for imaginary roots from the trace and the sum of the principal minors of matrix $(A)$.*

PROOF. From (A.13), we know that $3D^2 > (1/3)t^3$. Consequently, a when

$$t^2 + 4a_1 < 0, \qquad \text{(A.15)}$$

then the term inside the square root in (A.17) will be negative and the negative roots will be complex conjugates. Q.E.D.

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